Vehicle Trajectory Optimization at a Signalized Intersection in Mixed Traffic: Model and Field Experiments

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Keywords: Connected vehicle, automated vehicle, trajectory optimization, learning, field experiments

1. Introduction

Traffic flows on urban roads are regularly interrupted by traffic signals at intersections. As such, vehicles experience frequent stop-and-go movement and consume more fuel. Emerging connected & automated vehicle (CAV) technologies enable vehicles to receive the Signal Phase and Time (SPaT) information from the intersection and smooth their trajectories accordingly. However, trajectory optimization in mixed traffic is quite challenging due to uncertain human driver behaviors.

This study proposes a new vehicle trajectory optimization model to control a CAV in mixed traffic near a signalized intersection. The proposed model leverages learning methods and behavior models to predict downstream human-driven vehicles (HV) with SPaT and limited downstream traffic information. Based on the predicted information, a fast optimization algorithm is proposed to control the CAV to pass the intersection with a smooth trajectory while preserving the intersection throughput rate. The optimized CAV trajectory will consequently smooth the trajectories of following HVs in the mixed traffic. Finally, the key model development is validated with field experiments.

Our main contributions are as follows:

1) Propose a learning-based model to optimize CAV trajectories in mixed traffic at a signalized intersection.

2) Compare the performance of the two arrival time prediction methods (i.e., look-up table, and learning-based arrival time prediction).

3) Design and conduct field experiments to validate the proposed trajectory optimization model.

2. Methodology

The notations in Table 1 are used throughout this paper.

Table 1 Nation list.

Symbol	Description
m	Vehicle's driving mode. $m \in \mathcal{M} := \{\mathrm{C}, \mathrm{H}\}$. C means the vehicle is running as CAV, H means the vehicle is running as HV
$\mathcal{T}$	Time duration of the model
$\mathcal{N}$	Set of all vehicles
n	Vehicle number. $n \in \mathcal{N}$

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t	Time instant. $t \in \mathcal{T}$
$x_n^m(t)$	Location of vehicle n at time t, the coordinate is the distance from the stop line
$v_n^m(t)$	Speed of vehicle n at time t
$a_n^m(t)$	Acceleration of vehicle n at time t
$u_n^m(t)$	Jerk of vehicle n
L_{AB}	Distance between loop detector A and B
B_n	Mass of vehicle n
$t_n^{m\mathrm{A}}$	Time instant when vehicle n passes the loop detector A
$t_n^{m\mathrm{B}}$	Time instant when vehicle n passes the loop detector B
$\mathbf{X}_n^m(t)$	System state
$\dot{\mathbf{X}}_n^m(t)$	System dynamic function
$\mathcal{J}$	Total cost in the cost function
$R(v_n^m(t), a_n^m(t), u_n^m(t))$	Running cost
$S(\mathbf{X}_n(t_n^{m\mathrm{B}}))$	Terminal cost
$F_n^m(t)$	Instant fuel consumption of vehicle n
$\hat{x}_n^{\mathrm{B}}$	The desired position of vehicle n passing loop detector B (stop line)
$\hat{v}_n^{\mathrm{B}}$	The desired speed of vehicle n passing loop detector B (stop line)
$\hat{a}_n^{\mathrm{B}}$	The desired acceleration of vehicle n passing loop detector B (stop line)
v^{LIM}	Legal speed limit of the road
v^{MIN}	Minimum speed
a^{MAX}	Maximum acceleration
a^{MIN}	Minimum acceleration
u^{MAX}	Maximum jerk
u^{MIN}	Minimum jerk
K_1, K_2, K_3	Parameters of the terminal cost
$K_4, K_5, K_6, K_7, K_8, K_9, K_{10}$	Parameters of the running cost
K_{11}, K_{12}	Parameters of the linear car-following model
t_n^{MIN}	Earliest time of vehicle n passing the stop line without considering its preceding vehicle and signal control
t_n^{D}	Candidate travel time of vehicle n
t^{H}	Pre-set headway of two consecutive vehicles at the stop line
t_n^{G}	The start time of green light which is closest to t_n^{D}
$\mathrm{T}_k^{\mathrm{G}}$	The start time of green light in cycle k
$\mathrm{T}_k^{\mathrm{R}}$	The start rime of red light in cycle k
P^{R}	Duration of red phase
P^{G}	Duration of green phase
a_n^{ACC}	Target acceleration of vehicle n calculated from linear car-following model
τ_n^m	Desired time gap of vehicle n

2.1. Arrival time prediction

As CAV may not be the leading vehicle in the platoon, it is difficult to get accurate arrival time by mathematical model. As a result, look-up table is applied to predict the arrival time of the CAV. HVs are used to drive from loop detector A to detector B at different speed limit, and different SPaT information. After collecting and analyzing these data, a look-up table can be built. Four factors are considered in the look-up table, distance $x_n^{\mathrm{H}}(t)$, real-time speed $v_n^m(t)$, real-time headway $h^m(t)$, and SPaT information. After processing the accumulated data, the look-up table is able to predict the arrive time of CAV in real time.

$$t_n^{\mathrm{MIN}} = t_n^{m\mathrm{A}} + \frac{L_{\mathrm{AB}} - (\frac{v^{\mathrm{LIM}^2} - v_n^m(t_n^{m\mathrm{A}})^2}{2a^{\mathrm{MAX}}})}{v^{\mathrm{LIM}}} + \frac{v^{\mathrm{LIM}} - v_n^m(t_n^{m\mathrm{A}})}{a^{\mathrm{MAX}}} \tag{1}$$

$$t_n^{\mathrm{D}} = \max\{t_n^{\mathrm{MIN}}, t_{n-1}^{m\mathrm{B}} + t^{\mathrm{H}}\} \tag{2}$$

$$t_n^{\mathrm{G}} = \begin{cases} \mathrm{T}_k^{\mathrm{G}} & t_n^{\mathrm{D}} \in [\mathrm{T}_k^{\mathrm{G}}, \mathrm{T}_k^{\mathrm{R}}) \\ \mathrm{T}_{k+1}^{\mathrm{G}} & t_n^{\mathrm{D}} \in [\mathrm{T}_k^{\mathrm{R}}, \mathrm{T}_{k+1}^{\mathrm{G}}) \end{cases} \tag{3}$$

$$t_n^{m\mathrm{B}} = \max\{t_n^{\mathrm{G}}, t_n^{\mathrm{D}}\} \tag{4}$$

As to the learning-based arrival time prediction method, five factors, including distance $x_n^{\mathrm{H}}(t)$, real-time speed $v_n^m(t)$, real-time headway $h^m(t)$, SPaT information, and the real arrival time are sent into the convolutional neural network (CNN) for training. The training results are employed for real-time arrival time prediction during the trajectory optimization experiments.

2.2. Car-following model

The linearized car-following model is applied to model the HVs' driving behavior (Milanés et al. 2014).

$$a_n^{\mathrm{ACC}}(t) = K_{11}(x_{n-1}^m(t) - x_n^m(t) - v_n^m(t)\tau_n^m) + K_{12}(v_{n-1}^m(t) - v_n^m(t)) \tag{5}$$

2.3 Model predictive control

2.3.1. State variables

For an individual CAV indexed by n, the system state vector $\mathbf{X}_n^m(t)$ can be defined as follows:

$$\mathbf{X}_n^m(t) = [x_n^m(t), v_n^m(t), a_n^m(t)]^T \tag{6}$$

The state dynamics is shown as follows:

$$\dot{\mathbf{X}}_n^m(t) = f\big(x_n^m(t), v_n^m(t), a_n^m(t)\big) = [v_n^m(t), a_n^m(t), u_n^m(t)]^T \tag{7}$$

Where $\boldsymbol{a}_n^{\boldsymbol{m}}(t) = a_n^m(t)$ denotes the control input, which in this case is the acceleration of the vehicle n.

2.3.2. Objective function

The control goal is to drive AVs from the current position to the stop line with the desired velocity and acceleration.

$$\min_{a_n^m(t)} J = S\big(\mathbf{X}_n^m(t_n^{m\mathrm{B}})\big) + \int_{t_n^{m\mathrm{A}}}^{t_n^{m\mathrm{B}}} R(v_n^m(t), a_n^m(t), u_n^m(t))dt \tag{8}$$

Terminal cost:

$$S\big(\mathbf{X}_n^m(t_n^{m\mathrm{B}})\big) = K_1\big(x_n^m(t_n^{m\mathrm{B}}) - \hat{x}_n{}^{\mathrm{B}}\big)^2 + K_2\big(v_n^m(t_n^{m\mathrm{B}}) - \hat{v}_n{}^{\mathrm{B}}\big)^2 + K_3\big(a_n^m(t_n^{m\mathrm{B}}) - \hat{a}_n{}^{\mathrm{B}}\big)^2 \tag{9}$$

$$\hat{x}_n{}^{\mathrm{B}} = L_{\mathrm{AB}} \tag{10}$$

$$\hat{v}_n{}^{\mathrm{B}} = v^{\mathrm{LIM}} \tag{11}$$

$$\hat{a}_n{}^{\mathrm{B}} = 0 \tag{12}$$

Running cost:

$$R\left(v_n^m(t), a_n^m(t), u_n^m(t)\right) = K_4 + K_5 P^T\left(v_n^m(t), a_n^m(t)\right) + K_6 B_n v_n^m(t) a_n^m(t)^2 H\left(a_n^m(t)\right) + K_7 u_n^m(t)^2 \tag{13}$$

$$P^T\left(v_n^m(t), a_n^m(t)\right) = \max\{0, K_8 v_n^m(t) + K_9 v_n^m(t)^2 + K_{10} v_n^m(t)^3 + B_n v_n^m(t) a_n^m(t)\} \tag{14}$$

$$H\left(a_n^m(t)\right) = \begin{cases} 1 & a_n^m(t) > 0 \\ 0 & a_n^m(t) \le 0 \end{cases} \tag{15}$$

2.3.3. Constraints

Speed constraints: $v^{\mathrm{MIN}} \le v_n^m(t) \le v^{\mathrm{LIM}}$ (15)

Acceleration constraints: $a^{\mathrm{MIN}} \le a_n^m(t) \le a^{\mathrm{MAX}}$ (16)

Jerk constraints: $u^{\mathrm{MIN}} \le u_n^m(t) \le u^{\mathrm{MAX}}$ (17)

Safety constraints: $a_n^m(t) \le a_n^{\mathrm{ACC}}(t)$ (18)

3. Field experiments

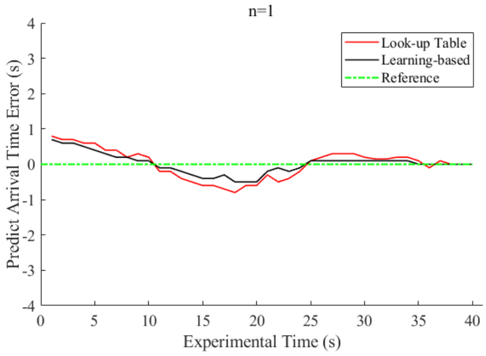
We conduct experiments on an approximate 400-meter straight road segment (Fig. 1). The loop detector is able to provide speed and time instant information when vehicle passes. Loop detector B is adjacent to the stop line and traffic light. The test track is furnished with DSRC RSU. All vehicles equipped with DSRC OBU running in the test track can receive SPaT information in real time. There are one CAV and four HVs used in the experiments.

Fig. 1 Satellite picture of the test track.

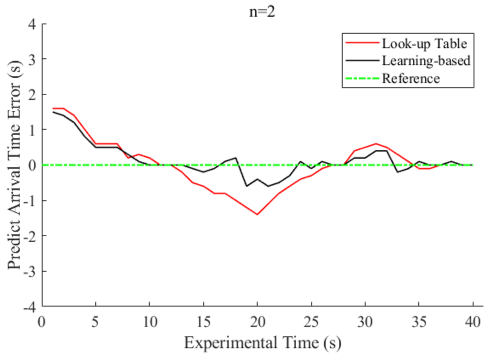
4. Results

4.1 Arrival time prediction

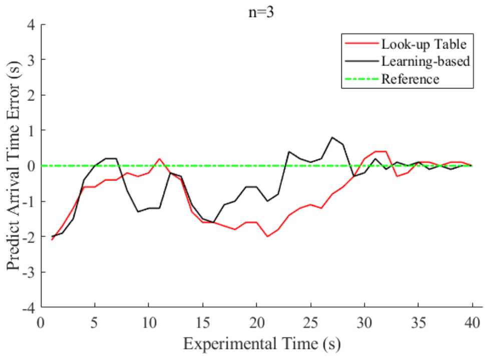
Arrival time prediction results of two methods are listed in Fig. 2. As can be seen in Fig. 2, the red lines and black lines correspond to the predicted arrival time based on the look-up table and learning-based methods, respectively. The learning-based arrival time prediction method achieves a smaller prediction error when the amount of human-driven vehicles before the CAV is less than four, $n < 4$. On the contrary, the look-up table arrival time prediction method is able to get a more stable prediction value; namely, the data vibration amplitude is very small.



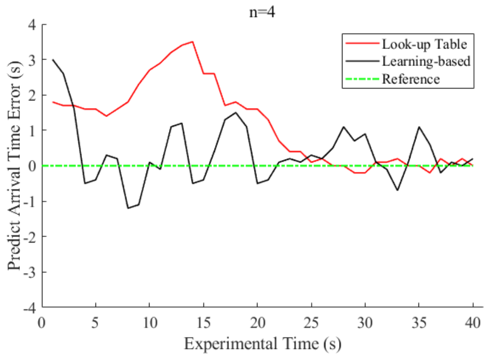
(a) n=1



(b) n=2



(c) n=3



(d) n=4

Fig. 2 Arrival time prediction results.

4.2 Trajectory optimization

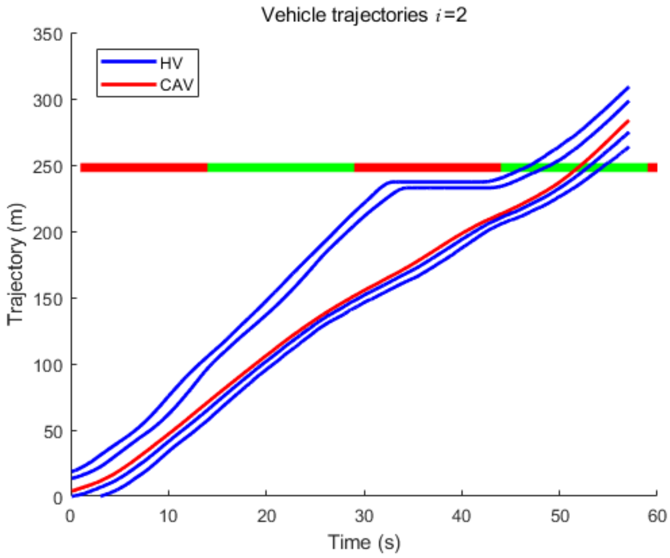


Fig. 3 Vehicle trajectories in the experiments (n=2).

One representative field experiment result is shown in Fig. 2. The read line is the trajectory of the CAV, and the blue lines are trajectories of the HVs. In this case, there are two HVs in front of the CAV in the platoon, there two HVs drive at a higher speed and come to a complete stop at the signalized intersection, the CAV and following HVs pass the intersection at the green phase without stop, and the speed trajectories and fuel consumption are all optimized.

5. Conclusion

Providing signal information to the vehicles on signalized urban roads is demonstrated to be an effective way to reduce the idle time and the fuel consumption. In this paper, a distributed and cooperative eco-driving method has been proposed for platoons to address these issues. The proposed eco-driving method has been designed for mixed traffic flow on an urban road, which consists of HVs and various penetrations of CAVs. The CAVs attempt to pass the intersection on the earliest possible green time with a maximum desired speed and zero acceleration.

References

Milanés, V., & Shladover, S. E. (2014). Modeling cooperative and autonomous adaptive cruise control dynamic responses using experimental data. Transportation Research Part C: Emerging Technologies, 48, 285-300.